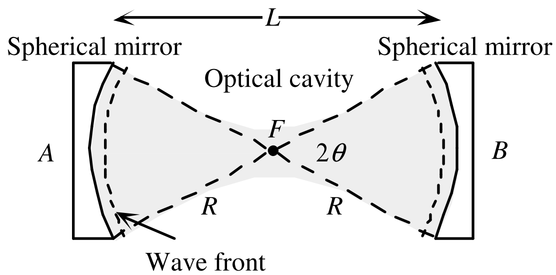
1.1 Gaussian beams Consider two identical spherical mirrors *A* and *B* that have been aligned to be focal directly face each other as in Figure 1Q1. The two mirrors and the space in between them (the optical cavity) form an **optical resonator** because only certain light waves with certain frequencies can exit in the optical cavity. The light beam in the cavity is a Gaussian beam. When it starts at *A* its wavefront is the same as the curvature of *A*. Sketch the wavefronts on this beam as it travels towards *B*, at *B*, as it is then reflected from *B* back to *A*. If *R* =25 cm, and the mirrors are of diameter 2.5 cm, estimate the divergence of the beam and its spot size (minimum waist) for light of wavelength 500 nm.



Two confocal spherical mirrors reflect waves to and from each other. *F* is the focal point and *R* is the radius. The optical cavity contains a Gaussian beam

Figure 1Q1

Solution

Let *D* = diameter of the mirror, from Figure 1Q1, $\tan\theta = (D/2)/R$ gives

$$\theta = \arctan(D/2R) = \arctan(0.05) = 0.05 \text{ rad.}$$

Divergence is 2θ or 0.1 rad.

Divergence 2θ and spot size $2w_o$ are related by

$$2\theta = \frac{4\lambda}{\pi(2w_o)}$$

and depends on the wavelength of interest. Taking $\lambda = 500$ nm,

and, $$2w_o = \frac{4\lambda}{\pi(2\theta)} = \frac{4(500\times10^{-9}\ \text{m})}{\pi(0.1)} = 6.4\times10^{-6}\ \text{m or about 6 micron.}$$

1.7 Phase changes on TIR Consider a light wave of wavelength 870 nm traveling in a semiconductor medium (GaAs) of refractive index 3.6. It is incident on a different semiconductor medium (AlGaAs) of refractive index 3.4, and the angle of incidence is 80°. Will this result in total internal reflection? Calculate the phase change in the parallel and perpendicular components of the reflected electric field.

Solution

a First calculate the critical angle: θ_c = arcsin(3.4/3.6) = 70.8°. The angle of incidence θ_i is greater than θ_c and hence there will be TIR. Since the incidence angle $\theta_i > \theta_c$ there is a phase shift in the reflected wave. The phase change in $E_{r,\perp}$ is given by $\phi_\perp$. With $n_1 = 3.6$, $n_2 = 3.4$ and $\theta_i = 80°$,

$$\tan\left(\tfrac{1}{2}\phi_\perp\right) = \frac{\left[\sin^2\theta_i - n^2\right]^{1/2}}{\cos\theta_i} = \frac{\left[\sin^2(80^\circ) - \left(\frac{3.4}{3.6}\right)^2\right]^{1/2}}{\cos(80^\circ)}$$

$$= 1.607 = \tan[\tfrac{1}{2}(116.21°)]$$

so that the phase change is

$\phi_\perp$ = **116.21°**.

For the $E_{r,//}$ component, the phase change is

$$\tan\left(\tfrac{1}{2}\phi_{//} + \tfrac{1}{2}\pi\right) = \frac{\left[\sin^2\theta_i - n^2\right]^{1/2}}{n^2\cos\theta_i} = \frac{1}{n^2}\tan\left(\tfrac{1}{2}\phi_\perp\right)$$

so that $\tan(\tfrac{1}{2}\phi_{//} + \tfrac{1}{2}\pi) = (n_1/n_2)^2\tan(\phi_\perp/2) = (3.6/3.4)^2\tan(\tfrac{1}{2}116.21°)$

which gives $\phi_{//}$ = **–58.1° (or 121.9°)**

We can repeat the calculation with θ_i = 90° to find $\phi_\perp$ = 180° and $\phi_{//}$ = 0°.

Note that as long as $\theta_i > \theta_c$, the magnitude of the reflection coefficients are unity. Only the phase changes.

For information: The amplitude of the evanescent wave as it penetrates into medium 2 is

$$E_{t,\perp}(y,t) \sim E_{to,\perp}\exp(-\alpha_2 y)$$

We ignore the z-dependence, $\exp j(\omega t - k_z z)$, as this only gives a propagating property along z. The field strength drops to $\boldsymbol{e}^{-1}$ when $y = 1/\alpha_2 = \delta$, which is called the **penetration depth**. The attenuation constant α_2 is

$$\alpha_2 = \frac{2\pi n_2}{\lambda}\left[\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1\right]^{1/2}$$

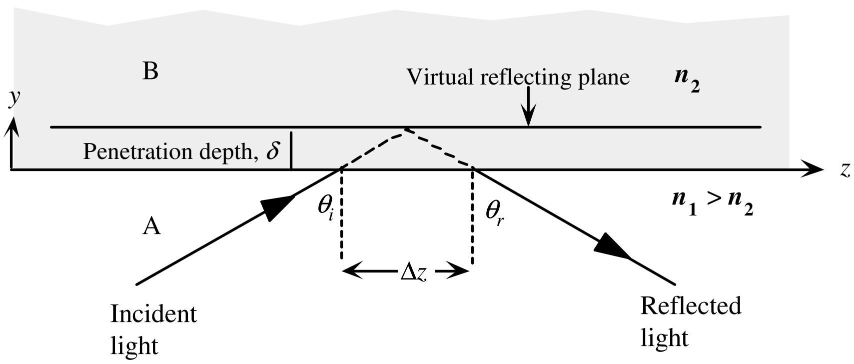
i.e.
$$\alpha_2 = \frac{2\pi(3.4)}{(870\times10^{-9}\ \text{m})}\left[\left(\frac{3.6}{3.4}\right)^2 \sin^2(80^\circ) - 1\right]^{1/2} = 7.26\times10^6\ \text{m}^{-1}.$$

so that the penetration depth is,

$$\delta = 1/\alpha_2 = 1/(7.26\times10^6\ \text{m}) = 1.38\times10^{-7}\ \text{m, or } 0.14\ \mu\text{m}.$$

1.12 Goos-Haenchen phase shift A ray of light which is traveling in a glass medium of refractive index $n_1 = 1.460$ becomes incident on a less dense glass medium of refractive index $n_2 = 1.430$. Suppose that the free space wavelength of the light ray is 850 nm. The angle of incidence $\theta_i = 85°$. Estimate the lateral Goos-Haenchen shift in the reflected wave for the perpendicular field component. Recalculate the Goos-Haenchen shift if the second medium has $n_2 = 1$ (air). What is your conclusion?

Solution



The reflected light beam in total internal reflection appears to have been laterally shifted by an amount Δz at the interface.

Figure 1Q12

The problem is shown in Figure 1Q12. When $\theta_i = 85°$,

$$\alpha_2 = \frac{2\pi n_2}{\lambda_o}\left[\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1\right]^{1/2}$$

$\therefore$

$$\alpha_2 = \frac{2\pi(1.430)}{(850\times10^{-9}\text{ m})}\left[\left(\frac{1.460}{1.430}\right)^2 \sin^2(85^\circ) - 1\right]^{1/2} = \mathbf{1.96\times10^6\ m^{-1}}$$

The penetration depth is $\delta = 1/\alpha_2 = 5.09\times10^{-7}$ m. The Goose-Haenchen shift is

$$\Delta z = 2d\tan\theta = 2(5.09\times10^{-7}\text{ m})(\tan 85°) = 11.6\times10^{-6}\text{ m} = \mathbf{11.6\ \mu m}$$

We can repeat the calculation using $n_2 = 1$ (air), then we find $\delta = 1/\alpha_2 = 1.28\times10^{-7}$ m, and $\Delta z = 2d\tan\theta = 2(1.28\times10^{-7}\text{ m})(\tan 85°) = 2.93\times10^{-6}$ m = **2.93 μm**. The shift is small when the refractive index difference is large. The wave penetrates more into the second medium when the refractive index difference is smaller.

1.15 Spectral widths

a Suppose that the frequency spectrum of a radiation emitted from a source has a central frequency υ_o and a spectral width $\Delta\upsilon$. The spectrum of this radiation in terms of wavelength will have a central wavelength λ_o and a spectral width $\Delta\lambda$. Clearly, $\lambda_o = c/\upsilon_o$. Since $\Delta\lambda << \lambda_o$ and $\Delta\upsilon << \upsilon_o$, using $\lambda = c/\upsilon$, show that, the line width $\Delta\lambda$ and hence the coherence length l_c are

$$\Delta\lambda = \Delta\upsilon\frac{\lambda_o}{\upsilon_o} = \Delta\upsilon\frac{\lambda_o^2}{c} \qquad \text{and} \qquad l_c = c\Delta t = \frac{\lambda_o^2}{\Delta\lambda}$$

b Calculate $\Delta\lambda$ for a lasing emission from a He-Ne laser that has λ_o = 632.8 nm and $\Delta\upsilon \approx$ 1.5 GHz.

Solution

Consider $\lambda = c/\upsilon$ and then differentiate this with respect to υ,

$$\frac{d\lambda}{d\upsilon} = c\frac{-1}{\upsilon^2}$$

Now substitute for c, $c = \lambda\upsilon$, to get

$$\frac{d\lambda}{d\upsilon} = -\frac{\lambda}{\upsilon}$$

The negative sign means that if λ *increases* by $d\lambda$, υ *decreases* by $d\upsilon$. We are interested in small "intervals" around central values. The spectral width, the wavelength interval, $\Delta\lambda$ is much smaller than the emission wavelength, or the center wavelength, λ_o. Similarly $\Delta\upsilon << \upsilon_o$. We do not need the negative sign as $\Delta\lambda$ and $\Delta\upsilon$ the intervals centered on λ_o and υ_o. $\Delta\lambda$ and $\Delta\upsilon$ are positive quantities.

$$\Delta\lambda = \Delta\upsilon\frac{\lambda_o}{\upsilon_o} = \Delta\upsilon\frac{\lambda_o^2}{c}$$

The coherence length l_c is determined by the temporal coherence time Δt which is determined by the frequency width $\Delta\upsilon$, and hence by $\Delta\lambda$,

$$l_c = c(\Delta t) = c\left(\frac{1}{\Delta\upsilon}\right) = c\left(\frac{\lambda_o^2}{c\Delta\lambda}\right) = \frac{\lambda_o^2}{\Delta\lambda}$$

Given λ = 632.8 nm, $\Delta\upsilon$ = 1.5 GHz. Thus,

$$\upsilon = c/\lambda = (3\times10^8 \text{ m/s})/(632.8\times10^{-9} \text{ m}) = 4.738\times10^{14} \text{ s}^{-1} \text{ or Hz.}$$

Thus, $$\Delta\lambda = \left|\Delta\upsilon\frac{\lambda}{\upsilon}\right| = (1.5\times10^9 \text{ Hz})\frac{(632.8\times10^{-9} \text{ m})}{(4.738\times10^{14} \text{ Hz})} = \mathbf{2.00\times10^{-12}\ m} \text{ or } \mathbf{2.00}$$

pm. 1.17 **Bragg Diffraction** Suppose that parallel grooves are etched on the surface of a semiconductor to act as a reflection grating and that the periodicity (separation) of the grooves is 1 micron. If light of wavelength 1.3 μm is incident at an angle 89° to the normal, find the diffraction beams.

Solution

Consider the Bragg diffraction grating equation,

$$d(\sin\theta_m - \sin\theta_i) = m\lambda \; ; m = 0, \pm1, \pm2, \ldots$$

Take m = 0 to find the zero-order diffraction, which is the normal reflected beam. The result is $\theta_m = \theta_I$ = 89° as shown in Fig1Q17; this identifies the meaning of the positive angle in the reflected beam with respect to the normal.

$$d(\sin\theta_m - \sin\theta_i) = m\lambda \ ; m = \ \pm 1, \pm 2, \ldots$$

Substituting $d = 1$ μm, $\lambda = 1.3$ μm, $\theta_i = 89°$, $m = -1$,

$$(1 \text{ μm})[\sin\theta_m - \sin(89°)] = (1)(1.3 \text{ μm})$$

solving, $\boldsymbol{\theta_m = -17.5°}$

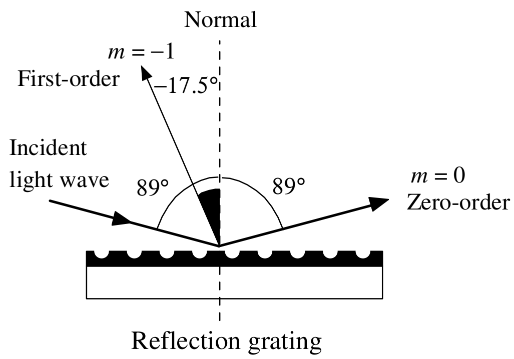


Figure 1Q17